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On Commutative and Medial of Bd-Algebras

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ABSTRACT

First of all, we begin this research by discussing the notion of B-algebras and d-algebras. Additionally, we provide some examples of it's algebraic structure. The idea of Bd-Algebras, which combines characteristics from B-algebras and d-algebras, is then explained. An algebraic structure $(X;*,0)$ is called Bd-algebras if for any $r,s \in X$ satisfies the following axioms: $r*0=r$, and if $r*s=0$ and $s*r=0$ then $r=s$. We also provide the example about Bd-algebras. Next, we look at the definitions of commutative and medial in Bd-algebras. After that, we investigated some proporsitions about commutative and medial Bd-algebras. Then, we proven thats propositions. As an example, if $(X;*,0)$ is a commutative and medial Bd-algebra, then

$$(r*u)*(s*v)=(v*u)*(s*r)$$

for any $r,s,u,v \in X$.

Keywords: B-algebras, d-algebras, Bd-algebras, commutative and medial Bd-algebras

1. INTRODUCTION

Mathematics has several branches, one of which is algebraic structure. Algebraic structures can also include BCK-algebras and BCI-algebras, in addition to groups, rings, or modules. BCK-algebras was created by Imai and Iseki [1] in 1966. Iseki introduced an original idea on BCI-algebras in the same year [2]. These algebras are a generalisation of BCK-algebras. Then, in 1999, Neggers and Kim created d-algebras, a new algebraic structure, by expanding the idea of BCK-algebras [3]. The ideas about fuzzy subalgebras and d-ideals in d-algebras are then presented by Akram and Dar [4]. They investigate into a few of their findings.

Not only that, 3 years later after [3], Neggers and Kim also introduced about B-algebra [5], which is an algebraic structure obtained from the properties of BCK-algebras and BCI-algebras. Some algebra also took several properties from BCI and BCH algebras [6]. In 2005, Kim and Park studied the concept of 0-commutative B-algebras [7]. After that in 2017, Jun et al. [8] introduced a new algebras from a B-algebra such as BH-algebra. Then, Saeid et

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al. [9] introduced new algebra namely BI-algebra too. Moreover, they compared some properties from one of algebra to other algebras such as $BCI/BCK/BCH/d/B$-algebras. One of the example is Multipolar Intuitionistic Fuzzy Ideal in B-Algebras by Amigo et al. [10]. They compared the properties in [11],[12],[13] to their novelty by using literary study and analogical related concepts. Recently, Amigo et al. also introduced about Multipolar Intuitionistic Fuzzy Positive Implicative Ideal in B-Algebras [14].

So, with combining the concepts of B-algebra and d-algebra, Bantaojai constructed a new algebraic structure called Bd-algebra [15]. This structure takes the second property of B-algebras X namely $r * 0 = r$, for all $r \in X$ and the third property of d-algebras X namely $r * s = 0$ and $s * r = 0$ imply $r = s$, for all $r, s \in X$. Bd-algebras has several properties such as edge, commutative, 0-commutative, associative and medial. The novelty of this research is to prove the theorems related to commutative and medial on Bd-algebras. In the first, we start with definitions of Bd-algebras which are helpful to our main results. Then, we also give the definitions of properties in Bd-algebras. In the first, we start with definitions of B-algebras, d-algebras, and Bd-algebras which are helpful to our main results. Then, we also give the definitions of properties in Bd-algebras.

Definition 1.1 [16] A non-empty set X contains 0 as the identity element (right) and binary operations $*$ according to the following axioms for all $r, s, t \in X$, is called a B-algebra.

(B1). $r * r = 0$.
(B2). $r * 0 = r$..
(B3). $(r * s) * t = r * \big(t * (0 * s)\big)$.

Example 1.2 [16] Let $X = \{0, m, n, p\}$ be a set with Cayley table as follows:

Table 1.1. Cayley table for $(X; * ,0)$

$*$	$\mathbf{0}$	$\boldsymbol{m}$	$\boldsymbol{n}$	$\boldsymbol{p}$
$\mathbf{0}$	0	0	n	n
$\boldsymbol{m}$	m	0	p	n
$\boldsymbol{n}$	n	n	0	0
$\boldsymbol{p}$	p	n	m	0

Then, $(X; * ,0)$ is a B-algebra.

Example 1.3 [10] Let $(\mathbb{R}^+ - \{0\}; * ,1)$ with $'' * ''$ be a binary operation of $\mathbb{R}^+ - \{0\}$ defined by

$$r * s = \frac{r}{s}.$$

Then, $(\mathbb{R}^+ - \{0\}; * ,1)$ is a B-algebra.

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Definition 1.4 [3] A non-empty set X contains 0 as the identity element (right) and binary operations $*$ according to the following axioms for all $r, s \in X$, is called a d-algebra.

(d1). $r * r = 0$.
(d2). $0 * r = 0$.
(d3). $r * s = 0$ and $s * r = 0$ imply $r = s$.

Example 1.5 [3] Let $X = \{0, m, n\}$ be a set with Cayley table as follows:

Table 1.2. Cayley table for $(X;*,0)$

$*$	$\mathbf{0}$	$\boldsymbol{m}$	$\boldsymbol{n}$
$\mathbf{0}$	0	0	0
$\boldsymbol{m}$	n	0	n
$\boldsymbol{n}$	m	m	0

Then, $(X;*,0)$ is a d-algebra.

Definition 1.6 [15] A non-empty set X contains 0 as the identity element (right) and binary operations $*$ according to the following axioms for all $r, s \in X$, is called a Bd-algebra.

(Bd1).$r * 0 = r$.
(Bd2).$r * s = 0$ and $s * r = 0$ imply $r = s$.

Example 1.7 [15] Let $X = \{0, m, n, p\}$ be a set with Cayley table as follows:

Table 1.3. Cayley table for $(X;*,0)$

$*$	$\mathbf{0}$	$\boldsymbol{m}$	$\boldsymbol{n}$	$\boldsymbol{p}$
$\mathbf{0}$	0	0	p	0
$\boldsymbol{m}$	m	0	n	p
$\boldsymbol{n}$	n	n	n	p
$\boldsymbol{p}$	p	n	n	p

Then, $(X;*,0)$ is a Bd-algebra.

Definition 1.8 [15] Let $(X;*,0)$ be a Bd-algebras. For all $r, s, t, u \in X$, then X is said to be
i. Edge if $r * X = \{0, r\}$ where $r * X = \{r * u | u \in X\}$.
ii. Commutative if $r * s = s * r$.

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iii. 0-commutative if $r * (0 * s) = s * (0 * r)$.
iv. Associative if $(r * s) * t = r * (s * t)$.
v. Medial if $(r * s) * (t * u) = (r * t) * (s * u)$.

Proposition 1.9 [15] If $(X;* ,0)$ be a Bd-algebras, then for all $r, s \in X$ satisfies

i. $0 * 0 = 0$.
ii. $r * 0 = 0 \rightarrow X = \{0\}$.
iii. $r * (0 * r) = 0, (0 * r) * r = 0 \rightarrow 0 * r = r * 0 = r$.
iv. $(r * s) * (s * r) = 0, (s * r) * (r * s) = 0 \leftrightarrow X \ is\ commutative$.
v. $X\ is\ commutative\ and\ r * s = 0 \rightarrow r = s$.

Proposition 1.10 [15] If $(X;* ,0)$ be a commutative Bd-algebras, then for all $r, s \in X$ satisfies

i. $r * s = (0 * s) * (0 * r)$.
ii. $0 * (0 * r) = r$.
iii. X is 0-commutative.

Theorem 1.11 [15] Assume $(X;* ,0)$ be a associative Bd-algebras. Then, X is commutative if and only if X is 0-commutative.

Proposition 1.12 [15] If $(X;* ,0)$ be a medial Bd-algebras, then for all $r, s, t \in X$ satisfies

i. $0 * (r * s) = (0 * r) * (0 * s)$.
ii. $(r * s) * t = (r * t) * s$.
iii. $r * (s * t) = (r * s) * (0 * t)$.
iv. If X associative satisfying $(B1)$, then X is B-algebras.

2. METHODS

Using analogy concepts, we obtain and investigated the related characterizes of commutative and medial in Bd-algebras. The procedure in this study was carried out as follows: we constructing some proporsitions of commutative and medial in Bd-algebras by using analogy concepts in [7] and then we proven thats proporsitions.

3. RESULTS AND DISCUSSION

In this section, we talk over some of the commutative and medial characteristics in Bd-algebras. We prove the propositions proposed in this section.

Proposition 3.1. *If $(X;* ,0)$ is a commutative and medial Bd-algebra, then*

$$(r * u) * (s * v) = (v * u) * (s * r)$$

for any $r, s, u, v \in X$.

Proof. For any $r, s, u, v \in X$, we obtain

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$$\begin{aligned}(r*u)*(s*v) &= (s*v)*(r*u) && \textit{Commutative } Bd-Algebras\\ &= (s*r)*(v*u) && \textit{Medial } Bd-Algebras\\ &= (v*u)*(s*r) && \textit{Commutative } Bd-Algebras\end{aligned}$$

■

Proposition 3.2. *If $(X;*,0)$ is a a commutative and medial Bd-algebra, then*

$$(r*t)*(s*t) = (t*s)*(t*r)$$

for any $r,s,t \in X$.

Proof. By using proposition 3.1, we have

$$\begin{aligned}(r*t)*(s*t) &= (t*t)*(s*r) && \textit{Proposition } 3.1\\ &= (t*s)*(t*r) && \textit{Medial } Bd-algebra\end{aligned}$$

■

Proposition 3.3. *If $(X;*,0)$ is a commutative and medial Bd-algebra, then*

$$(r*u)*s = (0*u)*(s*r)$$

for any $r,s,u \in X$.

Proof. If we let $v = 0$ in Proposition 3.1, then

$$\begin{aligned}(r*u)*s &= s*(r*u) && \textit{Commutative } Bd-Algebras\\ &= (s*r)*(0*u) && \textit{Medial } Bd-Algebras\\ &= (0*u)*(s*r) && \textit{Commutative } Bd-Algebras\end{aligned}$$

■

Proposition 3.4. *If $(X;*,0)$ is a commutative and medial Bd-algebra, then*

$$r*(s*v) = v*(s*r)$$

for any $r,s,v \in X$.

Proof. If we let $u = 0$ in proposition 3.1, then

$$\begin{aligned}r*(s*v) &= (s*v)*r && \textit{Commutative } Bd-Algebras\\ &= (s*v)*(r*0) && Bd2\\ &= (s*r)*(v*0) && \textit{Medial } Bd-Algebras\\ &= (s*r)*v && Bd1\\ &= v*(s*r) && \textit{Commutative } Bd-Algebras\end{aligned}$$

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Proposition 3.5. *If* $(X;*,0)$ *is a* 0*-commutative and medial* Bd*-algebra, then*

$$(r*s)*t=(r*t)*s$$

for any $r,s,t\in X$.

Proof. For any $r,s,t\in X$, we obtain

$$\begin{aligned}(r*s)*t&=t*(r*s) && Commutative\ Bd-Algebras\\ &=(t*r)*(0*s) && Medial\ Bd-Algebras\\ &=s*\big(0*(t*r)\big) && 0-Commutative\ Bd-Algebras\\ &=s*\big((0*t)*(0*r)\big) && Medial\ Bd-Algebras\\ &=s*(r*t) && Commutative\ Bd-Algebras\\ &=(r*t)*s && Commutative\ Bd-Algebras\end{aligned}$$

■

Proposition 3.6. *Every associative, commutative and medial* Bd*-algebra is* B*-algebra.*

Proof.
1. From $(Bd2)$ and commutative if we let $s=r$, we obtain

$$r*r=0$$

2. From $(Bd1)$ we have,

$$r*0=r$$

3. For any $r,s,t\in X$, we get

$$\begin{aligned}(r*s)*t&=r*(s*t) && Associative\ Bd-Algebras\\ &=(r*s)*(0*t) && Medial\ Bd-Algebras\\ &=(0*t)*(r*s) && Commutative\ Bd-Algebras\\ &=(0*r)*(t*s) && Medial\ Bd-Algebras\\ &=(r*0)*(t*s) && Commutative\ Bd-Algebras\\ &=r*(t*s) && Bd1\\ &=r*\big(t*(s*0)\big) && Bd1\\ &=r*\big(t*(0*s)\big) && Commutative\ Bd-Algebras\end{aligned}$$

■

CONCLUSION

In this paper, we study and prove several properties on commutative and medial in Bd-Algebras. Thus, we predict that future studies will be able to apply these features to additional algebraic structures, such as BF, BG, and B-Algebras. The next researchers also can applied the concept of multipolar intuitionistic fuzzy ideal in Bd-Algebras.

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