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Actuarial Present Value Analysis of The Due Temporary Annuity

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ABSTRACT

An n-year term life annuity is a number of interest-bearing payments paid by the policyholder during his/her life until death starting from the time of the contract agreement up to n years. In discrete life annuities, the policyholder can make a series of payments at the beginning of the period (due annuity) or the end of the period (immediate annuity). The researcher will analyze the value of the n-year term life annuity paid by the policyholder at the beginning of the payment period using the survival function approach on the Gompertz distribution. The value of the n-year term life annuity is influenced by the magnitude of the discount factor and the survival function on the Gompertz distribution. The researcher will set the age limit of the policyholder starting from 20 years to 40 years with an n-year contract agreement. Using the Maximum Likelihood Method (MLE), the researcher obtains the estimated values of the parameters B and c, respectively, $10^{-6} \leq B \leq 10^{-4}$ and $1{,}10 \leq c \leq 1{,}15$. Also, the researcher used interest rates that correspond to BI interest rates (period January 2023 to April 2024) of $5.75\%, 6\%,$ and 6.25%. The calculation results show that the higher the interest rate applied, the smaller the actuarial present value for a person aged (x) for a term of n years.

Keywords: discount factor, Gompertz distribution survival function, and life annuity value

1. INTRODUCTION

The amount of interest is compensation given by the borrower of capital to the owner of capital for the use of capital during a certain period of time [1]. The interest rate is used to measure the amount of interest given compared to the capital. For example, an investment of a certain amount of money with an interest rate in the form of money too. The amount of capital invested is called the principal and the total amount received after a certain period of time is called the accumulated value. Assume that during the period of time in question, there is no addition or reduction to the principal. Interest is the accumulated value minus the principal.

$$i = a(1) - a(0) \text{ or } a(1) = 1 + i$$

Present Value and Future Value Analysis

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At the beginning of the period, $t = 0$, the principal value of the investment is 1 [2]. One period later, $t = 1$, the investment value accumulates to $(1 + i)$. This is called the accumulated value or future value. Conversely, if the accumulated value at $t = 1$ is 1, then the present value is $(1 + i)^{-1} = v$. This value is called the discount factor. These two definitions are illustrated in the following figure:

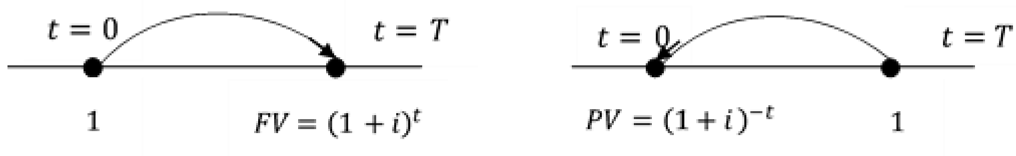

Figure 1.1. Future value (left) and present value (right)

In general, the accumulated value of 1 at time t is $(1 + i)^t$. Meanwhile, if the accumulated value at time t is 1, then the current value is $(1 + i)^{-t}$.

Survival Function

Suppose X is a continuous random variable that states the age until a person dies [3]. Define the distribution function of X as

$$F_X(x) = Pr(X \leq x), \qquad x \geq 0$$

which means the probability that a person will die before reaching the age of x years. And, define the survival function of X as the probability that a person will survive to the age of x years [4], namely:

$$S_X(x) = Pr(X > x), \qquad x \geq 0$$

or

$$S_X(x) = 1 - F_X(x), \qquad x \geq 0$$

Assume that the probability of a person being born and then dying at the age of 0 years is zero, namely $F_X(x) = 0$so that $S_X(0) = 1$ is obtained [5]. This means that the probability of a person being born and surviving to the age of 0 years is one.

Suppose (x) is a person's age when taking out life insurance, the remaining age (x) is $X - x$ denoted by $T(x)$ [6].The probability that a person aged (x) will die before reaching age $x + t$ is

$${}_tq_x = Pr(T(x) \leq t), \qquad t \geq 0,$$

while the probability that a person aged (x) years will survive to reach age x+t is

$${}_tp_x = Pr(T(x) > t) = 1 - {}_tq_x, \qquad t \geq 0$$

Force of Mortality

Let X be a continuous random variable that represents the age at which a person dies [7]. Define $\mu(x)$ as the death rate of a person aged (x) with

$$\mu(x) = \frac{f_X(x)}{S_X(x)} = \frac{-S_X{}'(x)}{S_X(x)}$$

for $\mu(x) \geq 0$. And, the relationship between ${}_tp_x$ and $\mu(x)$ can be expressed by:

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$$ {}_tp_x = \frac{S_X(x+t)}{S_X(x)} = \frac{e^{-\int_0^{x+t}\mu(y)dy}}{e^{-\int_0^{x}\mu(y)dy}} = e^{-\int_x^{x+t}\mu(y)dy} $$

where when $x = 0$ will result in:

$$ {}_t\boldsymbol{p}_{\boldsymbol{x}} = \boldsymbol{e}^{-\int_0^t \boldsymbol{\mu}(\boldsymbol{y})\boldsymbol{dy}} $$

Gompertz Distribution

The Gompertz distribution is a distribution introduced by Benjamin Gompertz, a British mathematician [8]. This distribution can be used to calculate the probability of life and the probability of death of a person aged (x). Also, researchers can calculate the approximate value of a person's life annuity aged (x) in a period of n years. The probability density function of the Gompertz distribution is

$$ f(x) = B \cdot c^x \cdot e^{\left\{\frac{-B}{\ln(c)}\cdot(c^x-1)\right\}}, \qquad 0 \le x \le \omega $$

with $B > 0, c > 1, x > 0$. Parameter B represents the force of mortality of a person aged (x), while parameter c represents the specific growth of the force of mortality of a person aged (x). Based on the probability density function, the cumulative function of the Gompertz distribution can be expressed by

$$ F(x) = 1 - e^{\left\{\frac{-B}{\ln(c)}\cdot(c^x-1)\right\}}, x \ge 0. $$

Meanwhile, the survival function is

$$ S(x) = e^{\left\{\frac{-B}{\ln(c)}\cdot(c^x-1)\right\}}, x \ge 0. $$

And the force of mortality of a person aged (x) can be expressed by:

$$ \mu(x) = B \cdot c^x. $$

A Life Annuity

A life annuity is a series of interest-bearing payments paid by the policyholder during his/her life until death based on an agreed period of time [9]. Life annuities are divided into three types, namely the whole life annuity, the temporary annuity, and the deferred whole life annuity[10] . Based on the payment period, life annuities are divided into two, namely the life annuity paid at the beginning of the period (due annuity) and the life annuity paid at the end of the period (immediate annuity). In this study, the researcher will calculate the life annuity whose fixed payments are made at the beginning of the period.

For example, the benefit payment paid at the beginning of the payment period each year is Rp1, - which is illustrated as follows:

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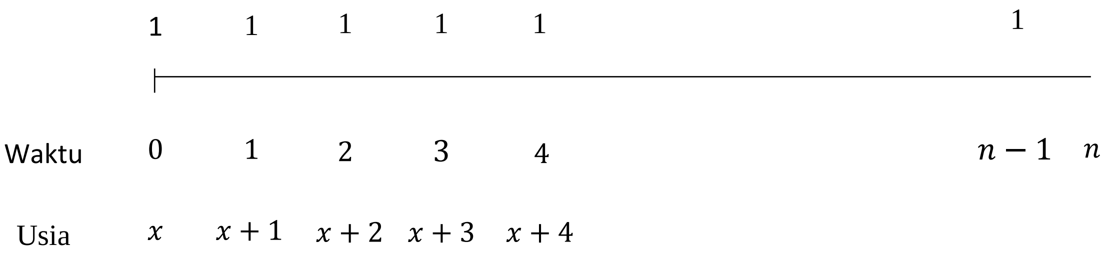

Figure 1.2. Illustration of Term Annuity Benefit Payments n years

The random variable benefit from the value of an n-year term life annuity with an annual payment of Rp1,- for a person aged x years who remains alive until n years later is [11]:

$$Y = \begin{cases} \ddot{a}_{\overline{k+1|}}, & 0 \leq K < n \\ \ddot{a}_{\overline{n|}}, & K \geq n \end{cases}$$

and the actuarial present value with payment at the beginning of the period for a person aged (x) and a term of n years is [12]

$$\ddot{a}_{x:\overline{n|}} = E[Y] = \sum_{k=0}^{n-1} \ddot{a}_{\overline{k+1|}} \cdot {}_{k}p_x \cdot q_{x+k} + \ddot{a}_{\overline{n|}} \cdot {}_{n}p_x$$

By adding each part where $\Delta f(k) = {}_{k}p_x \cdot q_{x+k} = {}_{k}p_x - {}_{k+1}p_x$ and $g(k) = \ddot{a}_{\overline{K+1|}}$. Using the relationship $\Delta g(k) = \Delta \ddot{a}_{\overline{k+1|}} = v^{k+1}$ and $f(k) = -{}_{k}p_x$ we get [13]:

$$\ddot{a}_{x:\overline{n|}} = \sum_{k=0}^{n-1} v^k \cdot {}_{k}p_x$$

2. METHODS

Based on the force of mortality in the Gompertz distribution, researchers will determine the probability of a person aged (x) who will survive to age $x + t$, ${}_{t}p_x$, using the following formula [14]:

$${}_{t}p_x = e^{-\int_x^{x+t} \mu(y)dy}$$

$$= exp\left(-B \cdot \frac{c^x}{\ln(c)} \cdot (c^t - 1)\right)$$

so that the function ${}_{t}p_x$ is obtained:

$${}_{t}p_x = g^{\{c^x \cdot (c^t - 1)\}}$$

The researcher will also determine the age of the policyholder starting from 20 years to 40 years with a contract agreement period of 5 years, 10 years, 15 years, 20 years, 25 years, 30 years, 35 years, and 40 years. In addition, the researcher will determine the interest rate according to the BI interest rate (period January 2023 to April 2024) of 5.75%, 6%, and 6.25%. The approach to the annuity value of a person's life aged (x) with a term of n years is influenced by the interest rate and the survival function of the

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Gompertz distribution which is stated by:

$$\ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^k \cdot g^{\{c^x \cdot (c^k - 1)\}}$$

Parameters B and c are estimated using the Maximum Likelihood (MLE) method [15]. Based on the probability density function of the Gompertz distribution, the researcher uses the likelihood equation to be

$$L(B,c) = \prod_{i=1}^{n} B \cdot c^x \cdot e^{\left\{\frac{-B}{\ln(c)} \cdot (c^x - 1)\right\}}.$$

Then, the researcher performs a natural logarithm on the likelihood function above to obtain

$$\ln L(B,c) = \ln\left[B^n \cdot c^{\sum_{i=1}^{n} x_i} \cdot e^{\left\{\frac{-B}{\ln(c)} \cdot \sum_{i=1}^{n}(c^{x_i} - 1)\right\}}\right]$$

$$= n \cdot \ln(B) + \sum_{i=1}^{n} x_i \cdot \ln(c) - \frac{B}{\ln(c)} \cdot \sum_{i=1}^{n} (c^{x_i} - 1)$$

The function $\ln L(B,c)$ is called the log-likelihood function. Maximize the log-likelihood function above with respect to parameters B and c as follows:

$$\frac{\partial \ln L(B,c)}{\partial B} = 0$$

and

$$\frac{\partial \ln L(B,c)}{\partial c} = 0$$

so that the form is obtained:

$$\frac{n}{B} - \frac{1}{\ln(c)} \cdot \sum_{i=1}^{n} (c^{x_i} - 1) = 0$$

and

$$\frac{1}{c} \cdot \sum_{i=1}^{n} x_i + \frac{B}{c \cdot \ln(c)^2} \cdot \sum_{i=1}^{n} (c^{x_i} - 1) - \frac{B}{\mathrm{c} \cdot \ln(c)} \cdot \sum_{i=1}^{n} x_i \cdot c^{x_i}.$$

The two maximum equations of the log-likelihood function above reflect the interdependent parameters. Therefore, the researcher uses the Newton Raphson method to estimate the parameters B and c. Choose the initial values of B_0 and c_0, respectively, are

$$\begin{pmatrix} B_0 \\ c_0 \end{pmatrix} = \begin{pmatrix} 0{,}0000001 \\ 1{,}10 \end{pmatrix}$$

where $10^{-6} \leq B \leq 10^{-4}$ dan $1{,}10 \leq c \leq 1{,}15$. With the help of R software, the estimated values of the parameters B and c of the Gompertz distribution, respectively, are $B = 0.0000006809$ and $c = 1.116$.

3. RESULTS AND DISCUSSION

The researcher will calculate the initial life annuity value of n years for a person aged (x) with a premium payment once a year using the Gompertz Distribution.

A. The First Case

A 20-year-old woman will buy a 10-year life insurance product with an interest rate of 6%

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(according to the BI rate for the period January 2023 to April 2024). Suppose the insurance company sets an annual premium for this woman of Rp2,000,000. Determine the initial life annuity value of n years.

Step one. The researcher will calculate the discount factor with the equation

$$v = \frac{1}{1+i} = \frac{1}{1+0{,}06} = 0{,}943396226.$$

Step two. The researcher will calculate the value of g by substituting the Gompertz distribution parameters, $B = 0.0000006809$ and $c = 1.116$, using the equation

$$g = 0{,}9999937960.$$

Step three. The researcher will calculate the initial life annuity value of 10 years for a 20-year-old woman with an annual premium payment of Rp1,- using the Gompertz distribution.

$$\ddot{a}_{20:\overline{10|}} = \sum_{k=0}^{10-1} v^k \cdot g^{\{1{,}116^{20} \cdot (1{,}116^k - 1)\}}$$
$$= \sum_{k=0}^{9} (0{,}943396226)^k \cdot (0{,}9999937960)^{\{1{,}116^{20} \cdot (1{,}116^k - 1)\}}$$
$$\ddot{a}_{20:\overline{10|}} = 7{,}8014168578$$

Step four. The researcher will calculate the initial life annuity value of 10 years for a 20-year-old woman with an annual premium payment of Rp2,000,000,- using the equation

$$P \cdot \ddot{a}_{20:\overline{10|}} = Rp2.000.000,- \cdot (7{,}8014168578) = Rp15.602.834,-$$

In general, the initial life annuity value of n years for a person aged (20) with an annual premium of Rp2,000,000,- and an interest rate of 6% is as follows:

Table 3.1. Initial Life Annuity Value of n Years for a Person Aged (20) with an Annual Premium of Rp2,000,000,- and an Interest Rate of 6% using the Gompertz Distribution

n	$\ddot{a}_{20:\overline{n|}}$
10	15,602,834
15	20,588,659
20	24,313,791
25	27,096,695
30	29,175,297
35	30,727,325
40	31,885,510

One of the purposes of a policyholder in purchasing life insurance products is to provide protection for himself in case of an accident. In this case, a person aged (20) buys a term life insurance product for n years with different terms. Based on Table 3.1, the longer the insurance term, the greater the annuity value. For $n = 20$, the policyholder buys an insurance product that provides self-protection for 20 years. This period is longer when compared to a period of 10 years or 15 years. As a result, the actuarial present value with payment at the beginning of the period for a person aged (x) with a period of 20 years is greater when compared to the actuarial present value of a term of 10 years or 15 years.

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B. The Second Case

With the same calculation steps, the researcher will analyze the initial life annuity value of n years for a person aged (x) with different interest rates, namely 5.75%, 6%, 6.25%.

Table 3.2. Initial Life Annuity Value of n Years for a Person Aged (x) with Annual Premium of Rp2,000,000,- and Interest Rate of 5.75% using Gompertz Distribution

n	$\ddot{a}_{20:\overline{n}|}$	$\ddot{a}_{25:\overline{n}|}$	$\ddot{a}_{30:\overline{n}|}$	$\ddot{a}_{35:\overline{n}|}$	$\ddot{a}_{40:\overline{n}|}$
10	15,752,074	15,751,665	15,750,958	15,749,733	15,747,613
15	20,879,813	20,878,834	20,877,139	20,874,206	20,869,129
20	24,756,476	24,754,635	24,751,449	24,745,934	24,736,395
25	27,686,978	27,683,921	27,678,633	27,669,483	27,653,661
30	29,901,817	29,897,105	29,888,954	29,874,858	29,850,500
35	31,575,210	31,568,284	31,556,307	31,535,609	31,499,886
40	32,838,792	32,828,936	32,811,902	32,782,501	32,731,857

Based on Table 3.2., the actuarial present value with payment at the beginning of the period for a person aged (x) with a term of n years with an interest rate of 5.75% is greater for longer n because the life protection period is longer.

Table 3.3. Initial Life Annuity Value of n Years for a Person aged (x) with an Annual Premium of Rp2,000,000 and an Interest Rate of 6% using the Gompertz Distribution

n	$\ddot{a}_{20:\overline{n}|}$	$\ddot{a}_{25:\overline{n}|}$	$\ddot{a}_{30:\overline{n}|}$	$\ddot{a}_{35:\overline{n}|}$	$\ddot{a}_{40:\overline{n}|}$
10	15,602,834	15,602,431	15,601,734	15,600,527	15,598,439
15	20,588,659	20,587,702	20,586,045	20,583,179	20,578,218
20	24,313,791	24,312,007	24,308,918	24,303,573	24,294,325
25	27,096,695	27,093,757	27,088,673	27,079,878	27,064,668
30	29,175,297	29,170,806	29,163,037	29,149,602	29,126,385
35	30,727,325	30,720,782	30,709,466	30,689,911	30,656,160
40	31,885,510	31,876,282	31,860,335	31,832,806	31,785,386

Based on Table 3.3., the actuarial present value with payment at the beginning of the period for a person aged (x) with a term of n years with an interest rate of 6% is greater for longer n because the life protection period is longer.

Table 3.4. Initial Life Annuity Value of n Years for a Person aged (x) with an Annual Premium of Rp2,000,000 and an Interest Rate of 6.25% using the Gompertz Distribution

n	$\ddot{a}_{20:\overline{n}|}$	$\ddot{a}_{25:\overline{n}|}$	$\ddot{a}_{30:\overline{n}|}$	$\ddot{a}_{35:\overline{n}|}$	$\ddot{a}_{40:\overline{n}|}$
10	15,456,050	15,455,654	15,454,967	15,453,778	15,451,721
15	20,304,263	20,303,328	20,301,709	20,298,907	20,294,058
20	23,884,164	23,882,434	23,879,439	23,874,257	23,865,291
25	26,527,255	26,524,430	26,519,542	26,511,084	26,496,459
30	28,478,312	28,474,030	28,466,623	28,453,813	28,431,677
35	29,918,049	29,911,864	29,901,169	29,882,685	29,850,781
40	30,979,858	30,971,214	30,956,274	30,930,483	30,886,054

Based on Table 3.4, the actuarial present value with payment at the beginning of the period for a person aged (x) with a term of n years with an interest rate of 6.25% is greater for longer n because the life protection period is longer.

CONCLUSION

The researcher will provide the results of the calculation of the actuarial present value of a person aged (x) with a term of n years as a consideration for the company in offering its products to prospective policyholders. Based on the calculation results above, the initial life annuity value of a person aged (x) with a term of n years is greater for longer terms n. This means that a policyholder will pay an increasingly large amount of money until the age of x+n years. Not only that, for increasingly large interest rates, the researcher concludes that the actuarial present value of a person aged (x) is smaller. With the list of annuity values, the researcher can provide an overview to prospective policyholders so that they can determine which term insurance product is more appropriate for them until the age of $x + n$.

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