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Threshold for Stop-Loss Reinsurance Modeling Using Expected Shortfall

[1]Lailatul Maziyah Wildan Mufaridho*, [2]Khaerun Nisa SH

[1]*Universitas Darunnajah, Jakarta, Indonesia*
[2]*Universitas Negeri Makassar, Indonesia*
*Email Corresponding author: lailatulmufaridho@darunnajah.ac.id

ABSTRACT
Insurance companies play a vital role in safeguarding individuals or groups from unforeseen risks. Reinsurance, particularly stop-loss reinsurance, is a key risk management strategy that provides insurers with protection against large claims. Setting an appropriate threshold in stop-loss reinsurance is crucial, as a low threshold increases reinsurance costs, while a high threshold heightens exposure to extreme losses. Traditional methods such as Value-at-Risk (VaR) are often used but have limitations in capturing heavy tail risks. Expected Shortfall (ES) offers a more robust alternative by accounting for both the probability and severity of losses beyond the threshold. This study explores the use of ES in determining stop-loss reinsurance thresholds. The proposed approach aims to improve risk management efficiency and strengthen the financial stability of insurers in the face of high uncertainty.

Keywords: Value-at-Risk, Expected-Shortfall, heavy tail

1. INTRODUCTION

An insurance company functions as a safeguard against unforeseen risks, both for individuals and groups. In conducting its business, the company faces various risks, such as the potential occurrence of large claims that could disrupt its financial condition. To manage these risks, insurance companies adopt a risk management strategy known as reinsurance. Reinsurance is an agreement in which part of the risk borne by the insurance company is transferred to a reinsurer. Consequently, the insurance company can reduce its financial burden in the event of significant claims.

One popular form of reinsurance is stop-loss reinsurance. Under this contract, the reinsurer covers claims that exceed a predetermined threshold agreed upon by both parties. The stop-loss contract provides significant benefits by offering protection against large claims, thereby shielding the insurance company from extreme financial losses.

Determining the threshold in a stop-loss contract is critical, as it directly impacts the financial stability of the insurance company. If the threshold is set too low, the insurer will frequently file claims with the reinsurer, leading to high premium costs payable to the reinsurer. This situation can reduce the insurer’s profitability. Conversely, if the threshold is set too high, the insurance company bears a greater risk, especially in the event of extreme claims. This condition could endanger the company's operational continuity.

Therefore, an appropriate method or approach is needed to determine the optimal threshold. Traditional approaches, such as Value-at-Risk (VaR), are commonly used in risk assessment. However, VaR has limitations, particularly in capturing fat-tail risks that may occur beyond the threshold. To address these shortcomings, Expected Shortfall (ES) has emerged as a more coherent alternative. ES not only considers the probability of risks exceeding the threshold but also accounts for the severity of losses occurring beyond that threshold [1][2]. This makes it a more reliable tool for measuring fat-tail risks in the context of reinsurance.

This study aims to analyze the use of Expected Shortfall in determining the threshold for stop-loss reinsurance. Through this approach, it is expected to develop a model that not only enhances risk management efficiency but also maximizes the financial stability of insurance companies amid high uncertainty.

2. METHODS

This study utilizes company expense data obtained from the Kaggle.com website, comprising 1,290 data points. The simulation is conducted using Google Colab. The steps involved in this research are as follows:

1. Fitting the distribution and parameter estimation

Insurance data often exhibit fat-tail characteristics. Therefore, this study will fit fat-tail distributions, such as Pareto and Cauchy. Parameter estimation will be performed using the Maximum Likelihood Estimation (MLE) method, which was first introduced by Sir Ronald Fisher in 1912 to derive estimators for unknown parameters [3][4][5].

Likelihood function $L(\theta)$ it is expressed as follow [6]

given a probability density function $f(r;\theta)$ where θ represents the parameter to be estimated, the likelihood function $L(\theta)$ for a sample $\{r_1, r_2, \ldots, r_n\}$ is defined as:

$$L(\theta) = \Pi_{i=1}^{n} f(r_i;\theta)$$

Taking the natural logarithm of the likelihood function, we obtain the log-likelihood function:

$$\ln L(\theta) = \sum_{i=1}^{n} \ln f(r_i;\theta)$$

The parameter $\hat{\theta}$ that maximizes the log-likelihood function is considered the MLE of θ. This is achieved by solving the equation:

$$(\partial \ln L(\theta))/\,(\partial\theta) = 0$$

2. Determining the threshold with Expected Shortfall

Value-at-Risk (VaR) is a commonly used risk measure. VaR is denoted as

$$VaR_\alpha(R) = \inf\{r \in \mathbb{R}: P[R \le r] \ge \alpha\}.$$

VaR has the drawback of being incoherent because it does not satisfy the subadditivity property [1][2]. Therefore, in this study, the risk measure Expected Shortfall (ES) will be used, which is expressed as follows [7][8].

$$ES_\alpha(R) = E[R|R > VaR_\alpha(R)$$

ES is the average that exceeds the VaR value and is coherent [8].

3. Application to the Stop-Loss Reinsurance Contract Model

Total risk is the aggregate of the risk borne by the insurance and the reinsurance. The risk borne by the insurance is denoted as R_I, while the risk borne by the reinsurance is denoted as R_R. Therefore, the total risk is $R = R_I + R_R$. According [3][9] Stop-Loss reinsurance contract is defined :

$$R_I = R \wedge d = \min\{R, d\} = \begin{cases} R\ , & R \le d \\ d, & R \ge d \end{cases}$$

Meanwhile, the risk borne by the reinsurance,

$$R_R = (R-d)_+ = \max\{0, R-d\} = \begin{cases} 0\ , & R \leq d \\ R-d, & R \geq d \end{cases}$$

Denoted by d, represent the retention value used in Stop-Loss. The value of d in this study will refer to the threshold determined bye ES.

3. RESULTS AND DISCUSSION

Table 1. Descriptive Statistics of Total Expenses

Me an	Std	Min	Ma x	Q_1	Q_2	Q_3
5.297	9.718	3.871	7.324	2.000	1.015	4.701

The Table 1 presents the descriptive statistics of Expenses (R). The mean of the data is greater than the median, indicating that the mean is located to the right of the median. As a result, the curve formed will exhibit a right skewness [10].

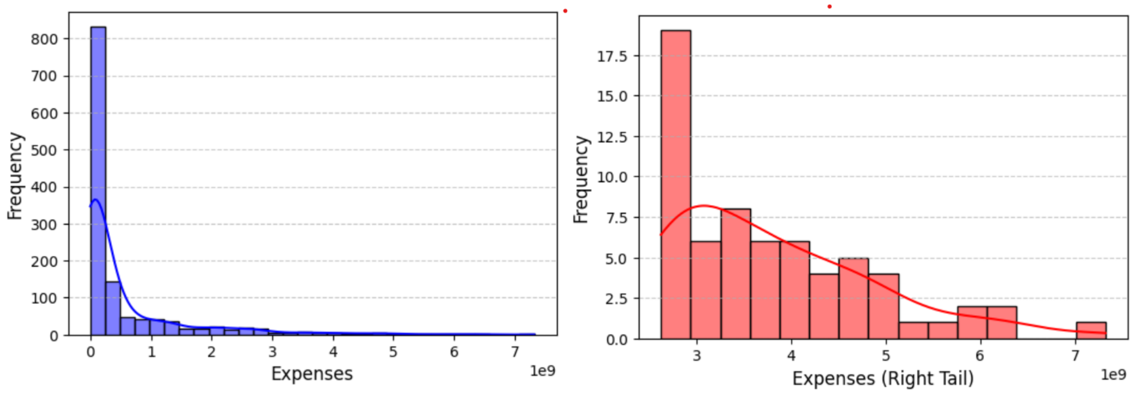

Figure 1. (a) Illustration of the expenses histogram (b) Illustration of the histogram for the tail expenses portion.

The Figure 1. Illustrates the total expanses, showing that the data exhibits a heavy tail. In illustration (b), the histogram of the right tail is shown. Therefore, the next step will be fitting the distribution. The candidate distribution for heavy tail data are Pareto and Cauchy.

Table 2. Distribution Fitting Results

Distribution (θ)	Log-Likelihood	AIC
Pareto	−26608.407	5322.816
Cauchy	−27466.133	54936.267

Based on the Table 2, it shows that the data is best modeled with the Pareto distribution, as it has the smallest log-likelihood and AIC values. Next, the Expected Shortfall (ES) value will be provided as the threshold for the expenditure amounts of the insurance and reinsurance. In Figure 2, it shows that the ES value for the data is 2044415152.05. This ES value will serve as the threshold for both insurance and reinsurance.

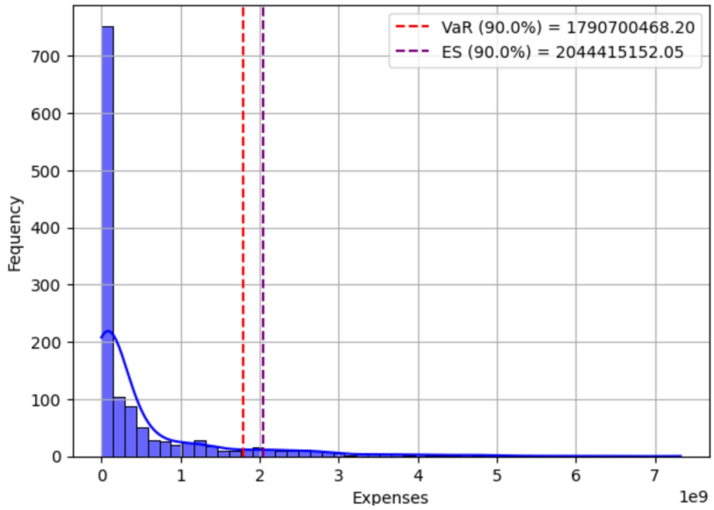

Figure 2. Illustration of the VaR and ES positions for expenses

The allocation of insurance expenses and reinsurance expenses using Expected Shortfall (ES) is as follows [9]

$$ES(R_I) = \begin{cases} ES(R) \ , & ES(R) \le d \\ d, & ES(R) \ge d \end{cases}$$

$$ES(R_R) = \begin{cases} 0 \ , & ES(R) \le d \\ ES(R) - d, & ES(R) \ge d \end{cases}$$

Next, an illustration of the expanses for insurance and reinsurance will be provided using the Stop-Loss contract model.

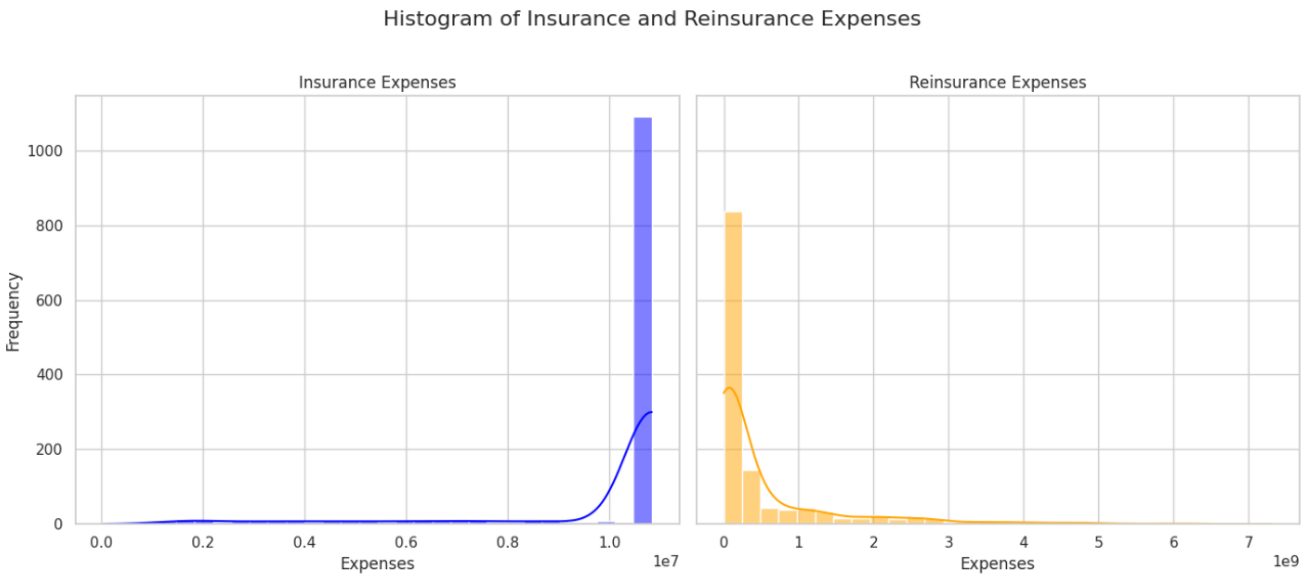

Figure 3. Illustration of insurance expenses and reinsurance expenses

Figure 3 is a histogram for insurance expenses and reinsurance expenses using the Stop-Loss reinsurance contract model. From the perspective of insurance, it is evident that using this type of contract is more advantageous for the insurance company, as it does not incur as large expenditures compared to reinsurance.

CONCLUSION

Determining the threshold using Expected Shortfall is more coherent than using Value-at-Risk. For the case of expense data (heavy-tail data) in this study, and from the insurance perspective, using a Stop-Loss contract is more advantageous, as the expenses borne by the insurance company are lower.

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