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Complete Ideal in Bd-Algebras

[1]Muhamad Hilman Rizaldi and [2]Royyan Amigo*

[1]Universitas Darunnajah, Jakarta, Indonesia
[2]Universitas Brawijaya, Malang, Indonesia

*Email Corresponding author: royyanamigo@darunnajah.ac.id

ABSTRACT

We start with the definition of B-algebras and d-algebras. Then, we give several examples of thats algebraic structure. We combine some properties from B-algebras and d-algebras. $(X;*,0)$ is called Bd-algebras if for any $r, s \in X$ satisfies the following axioms: $r * 0 = r$, and if $r * s = 0$ and $s * r = 0$ then $r = s$. We also explain the example of Bd-algebras. Next, we give the definition of ideal and complete ideal in Bd-algebras. We investigated and proven some proposition about thats concepts. One of the result in our research is every ideal is complete ideal in Bd-algebras.

Keywords: B-algebras, d-algebras, Bd-algebras, ideal, complete ideal

1. INTRODUCTION

In Mathematics, algebraic structure is the concept have many appeal to researchers. The example of algebraic structure is group, ring, module. Not only that, BCK-algebras and BCI-algebras are the example of algebraic structure too. BCK-algebras was introduced by Imai and Iseki [1] in 1966. In the same year, Iseki constructed an original idea about BCI-algebras [2]. These algebras are a generalisation of BCK-algebras. A few years later, Neggers and Kim created d-algebras, a new algebraic structure, by use the idea from BCK-algebras [3] in 1999. Then, the ideas about fuzzy subalgebras and d-ideals in d-algebras are explained by Akram and Dar [4]. They investigate some theorems and proposition about that.
Three years after [3], Neggers and Kim also introduced the concept of B-algebra [5], which is an algebraic structure combined the properties of BCK-algebras and BCI-algebras. Some researchers often combined several properties from some algebraic structure like BCI and BCH-algebras [6]. After that in 2017, Jun et al. [7] introduced a new algebras from a B-algebra such as BH-algebra. Then, Saeid et al. [8] introduced new algebra namely BI-algebra too. Moreover, they compared some properties from one of algebra to other algebras such as $BCI/BCK/BCH/d/B$-algebras.

Not only compared, the researchers constructed the new idea in algebraic structure. Kim and Park constructed the concept of 0-commutative B-algebras [9] in 2005. After that, many researchers studied about ideal in B-algebras. Abdullah and Atshan explained about complete ideal and n-ideal of B-algebras [10] or Fitria et al. explained about prime ideals in B-algebras [11] in 2017. Then, multipolar intuitionistic fuzzy ideal in B-Algebras was introduced by

Amigo et al. [12]. They compared the properties in [13],[14],[15] to their novelty by using literary study and analogical related concepts. Recently, Amigo et al. also introduced about Multipolar Intuitionistic Fuzzy Positive Implicative Ideal in B-Algebras [16].
By combined the concepts of B-algebra and d-algebra, Bantaojai constructed a new algebraic structure called Bd-algebra [17]. This structure use the second characterize of B-algebras X namely $r * 0 = r,$ for all $r \in X$ and the third characterize of d-algebras X namely $r * s = 0$ and $s * r = 0$ imply $r = s,$ for all $r, s \in X$. Bd-algebras have some properties such as edge, commutative, 0-commutative, associative and medial. Not only that, complete ideal is a new structure in Bd-algebras. The novelty of this research is to prove the propositions about complete ideal in Bd-algebras. In the first, we start with definitions of B-algebras, d-algebras, and ideal in Bd-algebras which are helpful to our main results. Then, we also give the definition of complete ideal in Bd-algebras. Then, we give the prove some propositions about that.

Definition 1.1 [11] A non-empty set X contains 0 as the identity element (right) and binary operations $*$ according to the following axioms for all $r, s, t \in X$, is called a B-algebra.

(B1). $r * r = 0$.
(B2). $r * 0 = r$..
(B3). $(r * s) * t = r * (t * (0 * s))$.

Example 1.2 [11] Let $X = \{0, m, n, p\}$ be a set with Cayley table as follows:

Table 1.1: Cayley table for $(X;*,0)$.

$*$	$\mathbf{0}$	$\boldsymbol{m}$	$\boldsymbol{n}$	$\boldsymbol{p}$
$\mathbf{0}$	0	0	n	n
$\boldsymbol{m}$	m	0	p	n
$\boldsymbol{n}$	n	n	0	0
$\boldsymbol{p}$	p	n	m	0

Then, $(X;*,0)$ is a B-algebra.

Example 1.3 [12] Let $(\mathbb{R}^+ - \{0\};*,1)$ with $'' * ''$ be a binary operation of $\mathbb{R}^+ - \{0\}$ defined by

$$r * s = \frac{r}{s}.$$

Then, $(\mathbb{R}^+ - \{0\};*,1)$ is a B-algebra.

Definition 1.4 [3] A non-empty set X contains 0 as the identity element (right) and binary operations $*$ according to the following axioms for all $r, s \in X$, is called a d-algebra.

(d1). $r * r = 0$.
(d2). $0 * r = 0$.

(d3). $r * s = 0$ and $s * r = 0$ imply $r = s$.

Example 1.5 [3] Let $X = \{0, m, n\}$ be a set with Cayley table as follows:

Table 1.2: Cayley table for $(X; * ,0)$.

$*$	**0**	$\boldsymbol{m}$	$\boldsymbol{n}$
0	0	0	0
$\boldsymbol{m}$	n	0	n
$\boldsymbol{n}$	m	m	0

Then, $(X; * ,0)$ is a d-algebra.

Definition 1.6 [17] A non-empty set X contains 0 as the identity element (right) and binary operations $*$ according to the following axioms for all $r, s \in X$, is called a Bd-algebra.

(Bd1). $r * 0 = r$.
(Bd2). $r * s = 0$ and $s * r = 0$ imply $r = s$.

Example 1.7 [17] Let $X = \{0, m, n, p\}$ be a set with Cayley table as follows:

Table 1.3: Cayley table for $(X; * ,0)$.

$*$	**0**	$\boldsymbol{m}$	$\boldsymbol{n}$	$\boldsymbol{p}$
0	0	0	p	0
$\boldsymbol{m}$	m	0	n	p
$\boldsymbol{n}$	n	n	n	p
$\boldsymbol{p}$	p	n	n	p

Then, $(X; * ,0)$ is a Bd-algebra.

Definition 1.8 [17] Let $(X; * ,0)$ be a Bd-algebras. For all $r, s, t, u \in X$, then X is said to be

i. Edge if $r * X = \{0, r\}$ where $r * X = \{r * u | u \in X\}$.
ii. Commutative if $r * s = s * r$.
iii. 0-commutative if $r * (0 * s) = s * (0 * r)$.
iv. Associative if $(r * s) * t = r * (s * t)$.
v. Medial if $(r * s) * (t * u) = (r * t) * (s * u)$.

Proposition 1.9 [17] If $(X;*,0)$ be a Bd-algebras, then for all $r,s \in X$ satisfies

i. $0*0=0.$
ii. $r*0=0 \rightarrow X=\{0\}.$
iii. $r*(0*r)=0,(0*r)*r=0 \rightarrow 0*r=r*0=r.$
iv. $(r*s)*(s*r)=0,(s*r)*(r*s)=0 \leftrightarrow X\ is\ commutative.$
v. $X\ is\ commutative\ and\ r*s=0 \rightarrow r=s.$

Proposition 1.10 [17] If $(X;*,0)$ be a commutative Bd-algebras, then for all $r,s \in X$ satisfies

i. $r*s=(0*s)*(0*r).$
ii. $0*(0*r)=r.$
iii. X is 0-commutative.

Theorem 1.11 [17] Assume $(X;*,0)$ be a associative Bd-algebras. Then, X is commutative if and only if X is 0-commutative.

Proposition 1.12 [17] If $(X;*,0)$ be a medial Bd-algebras, then for all $r,s,t \in X$ satisfies

i. $0*(r*s)=(0*r)*(0*s).$
ii. $(r*s)*t=(r*t)*s.$
iii. $r*(s*t)=(r*s)*(0*t).$
iv. If X associative satisfying $(B1)$, then X is B-algebras.

Definition 1.13 [17] Assume $(X;*,0)$ be a Bd-algebras. A non empty subset I of X is called subalgebra if it is satisfies

i. $0 \in I.$
ii. $\forall\ r,s \in I \rightarrow r*s \in I.$

Definition 1.14 [17] Assume $(X;*,0)$ be a Bd-algebras. A non empty subset I of X is called ideal if it is satisfies

i. $0 \in I.$
ii. $r*s \in I\ and\ s \in I\ \rightarrow r \in I.$
iii. $\forall\ r \in I\ and\ s \in X \rightarrow r*s \in I.$

Definition 1.15 [17] Assume $(X;*,0)$ be a Bd-algebras. A non empty subset I of X is called normal if it is satisfies

$$r*s \in I\ and\ t*u \in I\ \rightarrow (r*t)*(s*u) \in I$$

Proposition 1.16 [17] If $(X;*,0)$ is edge in Bd-algebras, then $\{0\}$ is a ideal of Bd-algebras.

2. METHODS

Using analogy concepts, we obtain and investigated the related characterizes of complete ideal in Bd-algebras. The procedure in this study was carried out as follows: we constructing the definition of complete ideal in Bd-algebras and then prove some propositions of thats by using analogy concepts in [10] and [17].

3. RESULTS AND DISCUSSION

In this section, we talk over about complete ideal in Bd-algebras. We prove the propositions proposed in this section.

Definition 3.1 Assume $(X;*,0)$ be a Bd-algebras. A non empty subset I of X is called complete ideal (c-ideal) if it is satisfies

i. $0 \in I$.
ii. $r * s \in I \; and \; \forall \; s \in I \; such \; that \; s \neq 0 \rightarrow r \in I$

Example 3.2 Let $X = \{0, m, n, p\}$ be a set with Cayley table as follows:

Table 3.1: Cayley table for $(X;*,0)$.

$*$	**0**	$\boldsymbol{m}$	$\boldsymbol{n}$	$\boldsymbol{p}$
0	0	m	m	m
$\boldsymbol{m}$	m	m	m	m
$\boldsymbol{n}$	n	n	n	n
$\boldsymbol{p}$	p	m	m	m

Then, $\{0, a, c\}$ is subalgebra, ideal, and c-ideal in Bd-algebras.

Definition 3.3 A c-ideal I of Bd-algebra X is said to be closed c-ideal if it is also subalgebra.

Example 3.4 In Eexample 3.2, $I = \{0, a, c\}$ is closed c-ideal.

Proposition 3.5 *Assume $(X;*,0)$ be a Bd-algebras. Every ideal is c-ideal in Bd-algebras.*

Proof. Let I be an ideal of X and let $r * s \in I \; and \; \forall s \in I \; such \; that \; s \neq 0$,
i. If $I = \{0\}$, then I is c-ideal.
ii. If $I \neq \{0\}$, then $\exists s \in I$, such that $s \neq 0 \; and \; r * s \in I$. Since I is ideal, then $r \in I$. So, I is c-ideal.

■

Corollary 3.6 The converse of proposition is not true in general, $I = \{0\}$ in Example 1.7 is c-ideal but not ideal, since $0 * n = p \notin I$.

Proposition 3.7 *Every c-ideal of Bd-algebra is subalgebra.*

Proof. It is directly from Definition 3.1.

■

Corollary 3.8 In general, the converse of proposition is not true. From Example 1.7, $I = \{0\}$ is subalgebra but not c-ideal, since $a * a = 0, a \notin I$.

Proposition 3.9 *If* $(X;*,0)$ *be a edge of* Bd*-algebra then {0} is c-ideal.*

Proof. It is directly from Proposition 1.16 and Proposition 3.5.

■

Corollary 3.10 The intersection of two complete ideals is complete ideal.

Example 3.11 From Example 3.2, $\mathrm{I} = \{0, \mathrm{a}, \mathrm{c}\}$ and $\mathrm{J} = \{0, \mathrm{b}\}$ are c-ideal, so $\mathrm{I} \cap \mathrm{J} = \{0\}$ is c-ideal.

CONCLUSION

In this paper, we study and prove several properties on complete ideal in Bd-Algebras. Thus, we predict that future studies will be able to develop the other ideal like n-ideal or prime ideal in Bd-Algebras. The next researchers also can applied the concept of multipolar intuitionistic fuzzy complete ideal in Bd-Algebras.

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