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Portfolio Optimization with Mean Variance Efficient Portfolio Method on Jakarta Islamic Indexed Stocks

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ABSTRACT

An investor will aim to reduce risk and increase the anticipated return on investment. When investing in stock portfolios, investors require the appropriate approach to ascertain the proportion of each stock so that the resulting portfolio is optimal. In this study, the stocks indexed by the Jakarta Islamic Index (JII) are optimized using the Mean Variance Efficient Portfolio (MVEP) method. The stocks GOTO, EXCL, ANTM, ITMG, and INDF are used as simulation data. The purpose of this study is to use the MVEP approach to determine the best portfolio from a combination of many stocks. The research results indicate that an optimum portfolio will be formed if the proportion for each stock is 26.53% for GOTO, 24.82% for ACES, 24.66% for ADRO, and 23.99% for TLKM. With those proportions, the expected portfolio return is 0.0074%. The result is expected to serve as a basis for decision-making for investors when investing.

Keywords: mean variance, portfolio, stocks

1. INTRODUCTION

Investing is a type of commitment to a large amount of money that is made now with the intention of generating profits in the future and minimizing risk. Risk itself can be defined as the degree of uncertainty about the likelihood that a particular event will occur or that a failure will occur in achieving a particular goal within a given time frame [1]. Investing in securities form can be done through the modal or money market. Generally speaking, investors make modal market investments with the hope of earning a profit. According to the profitability metric, the risk of investing in the capital market is significantly higher than the risk of investing in the money market, such as deposits [2]. Investors can use saham portfolio strategies to reduce the risk of making saham investments.

One of the strategies that can be applied to minimize risk in investments is through diversification, which involves forming a stock portfolio consisting of a combination of several stocks owned by the investor. When investing in a portfolio, investors will strive to maximize the expected return and seek the lowest level of risk. A portfolio that has such characteristics is called an efficient portfolio.

When creating an efficient portfolio, investors typically steer clear of risk (risk aversion), which means that if two investments offer the same return but differ in risk, the investor would select the one with the lowest amount of risk. The most optimal portfolio is the one that an investor will select if their portfolio is efficient [3]. A portfolio with the highest predicted return and the lowest degree of risk is said to be efficient. Investors will select the most optimal portfolio if there are multiple options for efficient portfolios.

Among the methods often used to find an optimal portfolio is the Mean Variance Efficient Portfolio (MVEP) method. This method focuses on investments that only involve risky assets, without including risk-free assets in the portfolio. Several studies using the MVEP method have been conducted previously. Among them are Fauziah [4] who applied the MVEP method for bond portfolio optimization, Sanggup [1] who implemented the MVEP method for stock portfolio optimization, and Purba who used the Asset Pricing Model (CAPM) and the MVEP method for stock portfolio

optimization. Meanwhile, this research focuses more on the implementation of the MVEP method for optimizing stock portfolios indexed by the Jakarta Islamic Index (JII).

This study aims to optimize the investment value of a stock portfolio indexed by JII through the combination of several stocks. The research is expected to provide solutions in finding the optimal proportion for the portfolio in each of these sectors, thus becoming one of the references for investors decision making.

2. METHODS

The data used in this study is secondary data, specifically the closing stock prices of four stocks indexed in the Jakarta Islamic Index (JII) for the period from July 1 to November 29, 2024. The four companies are GOTO, ACES, ADRO, and TLKM. The closing stock prices used for the four companies were taken from the website www.yahoofinance.com [5].
The method used in this study is the *Mean Variance Efficient Portfolio* method, which is used to optimize the portfolio. The following will explain the terms and steps to implement the method.

2.1. Stocks Return

Stock return is the level of profit obtained by investors from owning shares of a company [6]. Stock return can be calculated by determining the difference between the purchase price and the selling price of the shares between two consecutive periods, then dividing it by the earlier period. Mathematically, stock return can be calculated using the following Eq. (1) [7].

$$R_t = \frac{S_t - S_{t-1}}{S_{t-1}} \tag{1}$$

where R_t denotes the stock return at time t, S_t is the stock closing price at time t, and S_{t-1} is the stock closing price at time t.

2.2. Portfolio Return

An investment that includes multiple stocks is called a portfolio. The percentage of an investor's investment in each asset is determined by the portfolio [8]. Eq. (2) is the formula used to determine a portfolio's expected return.

$$ER_p = \sum_{i=1}^{N} w_i R_i \tag{2}$$

where ER_p denotes the portfolio return, w_i is the weight of stock i, R_i is the return of stock i, and N is the number of stocks.

2.3. Jakarta Islamic Index

The Jakarta Islamic Index (JII) is a sharia stock index which consists of the 30 most liquid sharia shares listed on BER. Reviews Sharia shares are carried out in May and November each year accordingly schedule by Otoritas Jasa Keuangan (OJK) [9].

2.4. Mean Variance Efficient Portfolio (MVEP)

The basic principle of MVEP is how to form a portfolio with minimum variance among several possible portfolios that can be formed. Mathematically, this is equivalent to how to optimize the weight $w = (w_1,\ w_2, \dots, w_p)^T$ by minimizing the variance.

Before discussing MVEP further, let's first define some basic concepts, namely expectation (mean), variance, and covariance.

Definition 1. *The expectation of a dataset is defined as the sum of all observed data divided by many data* [10].

Suppose $x_1, x_2, \dots, x_n$ is a set of data observed on a sample of n sizes, then the expected value of the data can be expressed in Eq. (3).

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{3}$$

Definition 2: *The variance of a data group is defined as the mean of the square of the deviation of the observed values against the mean values.*

Sample variance is defined by following Eq. (4).

$$Var\ (X) = \frac{\sum(x_i - \bar{x})^2}{n-1} \tag{4}$$

Meanwhile, the combined variance between two variables, called a covariance, is defined following the Eq. (5)

$$Cov\ (X,Y) = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{n-1} \tag{5}$$

Definition 3: *Suppose X_i and X_j are random variables with $i = (1, 2, \dots, N)$ and $j = (1, 2, \dots, N)$ then the covariance of X_i and X_j notated with $\sum$ is defined according to* Eq. (6) [11].

$$\sum = \begin{pmatrix} Var(X_1) & Cov(X_1,X_2) & \cdots & Cov(X_1,X_N) \\ Cov(X_1,X_2) & Var(X_2) & \cdots & Cov(X_1,X_N) \\ \vdots & & \ddots & \vdots \\ Cov(X_1,X_N) & Cov(X_2,X_N) & \cdots & Var(X_N) \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\ \vdots & & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N1} & \cdots & \sigma_{NN} \end{pmatrix} \tag{6}$$

Next we will look for the weight vector $\boldsymbol{w}$ so that the portfolio formed is maximum with the constraints, namely as follows:

a. The initial specification of mean return μ_p must satisfy $\boldsymbol{w^T \mu}$
b. The total proportion of the portfolio formed must satisfy $\boldsymbol{w^T 1_N = 1}$ where $\mathbf{1}_N$ is a unit vector with dimensions $N \times 1$

The above optimization problem will be solved with the Lagrange function which satisfies Eq. (7).

$$L = \boldsymbol{w}^T \sum \boldsymbol{w} + \lambda_1\left(\mu_p - \boldsymbol{w}^T \mu_p\right) + \lambda_2(1 - \boldsymbol{w}^T \mathbf{1}_N) \tag{7}$$

where λ is the Lagrange multiplier factor.

Next, derive Eq. (7) regarding $\boldsymbol{w}$ to obtain the optimal solution $\boldsymbol{w}$ such that it satisfies Eq. (8)

$$\begin{gathered} \frac{\delta L}{\delta \boldsymbol{w}} = 0 \\ \frac{\delta}{\delta \boldsymbol{w}}\left[\boldsymbol{w}^T \sum \boldsymbol{w} + \lambda_1\left(\mu_p - \boldsymbol{w}^T \mu_p\right) + \lambda_2(1 - \boldsymbol{w}^T \mathbf{1}_N)\right] = 0 \end{gathered} \tag{8}$$

By solving the optimization problem above, we will obtain weightings that satisfy Eq. (9).

$$\boldsymbol{w} = \frac{\sum^{-1} \mathbf{1}_N}{\mathbf{1}_N{}^T \sum^{-1} \mathbf{1}_N} \tag{9}$$

where $\boldsymbol{w}^T$ is the transpose of the weight matrix and $\sum^{-1}$ is the inverse of the covariance matrix [1].

2.5. Summary of Study steps

This research will be carried out through the following steps.

a. Select stock data from JII indexed companies
b. Calculating stock returns using Eq. (1)
c. Building matrix of stock return covariance, namely $\sum$ using Eq. (3), Eq. (4), Eq. (5), and Eq. (6).
d. Determine the inverse covariance matrix $\sum^{-1}$ using R Studio
e. Determine the optimum portfolio weight matrix using equation Eq. (9)
f. Calculating expected return using Eq. (2)

3. RESULTS AND DISCUSSION

After selecting several stokcs indexed by JII, the stock return will then be determined using equation (1) and the expected stock return using Eq. (3). The results can be seen in Table 1.

Table 1. Stock returns

t	GOTO	ACES	ADRO	TLKM
1	0.0000	-0.0175	0.0410	-0.0162
2	0.0000	-0.0179	-0.0251	-0.0033
3	0.0000	0.0121	0.0184	-0.0231
105	-0.0263	-0.0064	0.0040	-0.0109
106	-0.0270	-0.0064	-0.0039	0.0037
107	-0.0139	-0.0065	-0.0316	-0.0037

Next is to determine the stock return covariance matrix. To build a covariance matrix from stock returns, the variance and covariance will first be determined using Eq. (4) and Eq. (5). After that, allthe variances and covariance that have been obtained are used as entries in the covariance matrix so that they satisfy Eq. (6) which results in Eq. (11).

$$\sum = \begin{pmatrix} 0.00093 & 0.00930 & 0.00932 & 0.00931 \\ 0.00930 & 0.00053 & 0.00926 & 0.00918 \\ 0.00932 & 0.00926 & 0.00047 & 0.00916 \\ 0.00931 & 0.00918 & 0.00916 & 0.00033 \end{pmatrix} \tag{11}$$

The covariance matrix, $\sum$ in equation (9) is then determined by its inverse using Microsost Excel, the results of which can be seen in equation (12)

$$\sum^{-1} = \begin{pmatrix} -77.71 & 39.03 & 38.48 & 37.66 \\ 39.03 & -77.61 & 36.78 & 36.83 \\ 38.48 & 36.78 & -77.29 & 36.83 \\ 37.66 & 36.83 & 36.83 & -77.47 \end{pmatrix} \tag{12}$$

Next, we will look for the weight or proportion of each stock. By using equation (8), the optimum portfolio weight vector is obtained in equation (13).

$$\boldsymbol{w} = \frac{\begin{pmatrix} -77.71 & 39.03 & 38.48 & 37.66 \\ 39.03 & -77.61 & 36.78 & 36.83 \\ 38.48 & 36.78 & -77.29 & 36.83 \\ 37.66 & 36.83 & 36.83 & -77.47 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}}{(1 \quad 1 \quad 1 \quad 1)\begin{pmatrix} -77.71 & 39.03 & 38.48 & 37.66 \\ 39.03 & -77.61 & 36.78 & 36.83 \\ 38.48 & 36.78 & -77.29 & 36.83 \\ 37.66 & 36.83 & 36.83 & -77.47 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}} = \begin{pmatrix} 0.2653 \\ 0.2482 \\ 0.2466 \\ 0.2399 \end{pmatrix} \tag{13}$$

Based on equation (13), the optimum portfolio based on the *Mean Variance Efficinet Portfolio* (MVEP) method can be achieved when the weight for each stock is 26.53% for GOTO, 24.82% for ACES, 24.66% for ADRO, and 23.99% for TLKM. The weight values that have been obtained are then used to determine the portfolio's expected return. By using equation (2), the expected portfolio return value is $4{,}12 \times 10^{-4}$. This is the most optimum stock return value based on the MVEP method.

CONCLUSION

Based on the results and discussion, it can be concluded that one method of stock portfolio optimization is the Mean Variance Efficient Portfolio (MVEP) method. This method can be used to determine the optimum portfolio by calculating the proportion of each share following equation (8). Based on the calculation results, the proportion of each share is obtained, namely stock worth 26.53% for GOTO, 24.82% for ACES, 24.66% for ADRO, and 23.99% for TLKM.

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